Statistical arbitrage and pairs trading

Nikos S. Thomaidis, PhD

Dept. of Financial Engineering & Management
University of the Aegean, GREECE

email: nthomaid@fme.aegean.gr
Dept URL: http://labs.fme.aegean.gr/decision/
Personal web site: http://users.otenet.gr/~nths18

\(^1\)in collaboration with Nicholas Kondakis, Kepler Asset Management LLC and NGSQ International Ltd, NY.
Outline

- What is pairs trading?
Outline

- What is pairs trading?
- Developing a pairs trading system from scratch
Outline

- What is pairs trading?
- Developing a pairs trading system from scratch
- Empirical study: statistical arbitrage between Dow Jones Industrial Average (DJIA) stocks
Outline

- What is pairs trading?
- Developing a pairs trading system from scratch
- Empirical study: statistical arbitrage between Dow Jones Industrial Average (DJIA) stocks
- Conclusions
Outline

- What is pairs trading?
- Developing a pairs trading system from scratch
- Empirical study: statistical arbitrage between Dow Jones Industrial Average (DJIA) stocks
- Conclusions
  - Trading risks
Outline

- What is pairs trading?
- Developing a pairs trading system from scratch
- Empirical study: statistical arbitrage between Dow Jones Industrial Average (DJIA) stocks
- Conclusions
  - Trading risks
  - Opportunities
Outline

- What is pairs trading?
- Developing a pairs trading system from scratch
- Empirical study: statistical arbitrage between Dow Jones Industrial Average (DJIA) stocks
- Conclusions
  - Trading risks
  - Opportunities
  - Future challenges

Pairs trading: the history

- Pairs trading is one of the most popular investment strategies.

Pairs trading: the history

- Pairs trading is one of the most popular investment strategies.
- Already in the mid 80’s, Morgan Stanley - and perhaps other investment companies - have started developing computer programs indicating long and short trading positions in the market\(^2\).

Pairs trading is one of the most popular investment strategies. Already in the mid 80’s, Morgan Stanley - and perhaps other investment companies - have started developing computer programs indicating long and short trading positions in the market\(^2\).

These strategies have been strongly quantitative/mechanical in nature (trading signals are generated using statistical/mathematical techniques, trades are executed by computer programmes).

Pairs trading: the history

- Pairs trading is one of the most popular investment strategies.
- Already in the mid 80’s, Morgan Stanley - and perhaps other investment companies - have started developing computer programs indicating long and short trading positions in the market\(^2\).
- These strategies have been strongly quantitative/mechanical in nature (trading signals are generated using statistical/mathematical techniques, trades are executed by computer programmes).
- The development of a pairs trading system typically involves the cooperation of experts with diverse background (mathematicians, statisticians, computer scientists and finance experts).

Pairs trading: main idea

See e.g. [Gatev et al., 2006].
Identify two securities with similar historical price trajectories.
Pairs trading: main idea\textsuperscript{a}

\textsuperscript{a}See e.g. [Gatev et al., 2006].

- Identify two securities with similar historical price trajectories.
- If at some point in time the relative price distance (\textit{spread}) exceeds a threshold, simultaneously long the undervalued security and short the overvalued one.
Pairs trading: main idea

See e.g. [Gatev et al., 2006].

- Identify two securities with similar historical price trajectories.
- If at some point in time the relative price distance *(spread)* exceeds a threshold, simultaneously long the undervalued security and short the overvalued one.
- This joint bet will generate profit if the spread closes again in the near future.
So what is pairs trading?
So what is pairs trading?

- a market-neutral trading strategy: its returns are uncorrelated with market movements.
So what is pairs trading?

- a market-neutral trading strategy: its returns are uncorrelated with market movements.
- a statistical arbitrage trading strategy: profits from temporal mispricings of an asset relative to its fundamental value.
So what is pairs trading?

- a market-neutral trading strategy: its returns are uncorrelated with market movements.
- a statistical arbitrage trading strategy: profits from temporal mispricings of an asset relative to its fundamental value.
- a long/short strategy: reduces exposure to systematic shocks by simultaneously going long and short in fundamentally related securities.
So what is pairs trading?

- **a market-neutral trading strategy**: its returns are uncorrelated with market movements.
- **a statistical arbitrage trading strategy**: profits from temporal mispricings of an asset relative to its *fundamental value*.
- **a long/short strategy**: reduces exposure to systematic shocks by simultaneously going long and short in fundamentally related securities.
- **relative-value trading**,
So what is pairs trading?

- **a market-neutral trading strategy**: its returns are uncorrelated with market movements.
- **a statistical arbitrage trading strategy**: profits from temporal mispricings of an asset relative to its *fundamental value*.
- **a long/short strategy**: reduces exposure to systematic shocks by simultaneously going long and short in fundamentally related securities.
- **relative-value trading, convergence trading**, 
So what is pairs trading?

• **a market-neutral trading strategy**: its returns are uncorrelated with market movements.

• **a statistical arbitrage trading strategy**: profits from temporal mispricings of an asset relative to its *fundamental value*.

• **a long/short strategy**: reduces exposure to systematic shocks by simultaneously going long and short in fundamentally related securities.

• **relative-value trading, convergence trading**, and so on...
So what is pairs trading?

- **a market-neutral trading strategy**: its returns are uncorrelated with market movements.
- **a statistical arbitrage trading strategy**: profits from temporal mispricings of an asset relative to its fundamental value.
- **a long/short strategy**: reduces exposure to systematic shocks by simultaneously going long and short in fundamentally related securities.
- **relative-value trading**, **convergence trading**, and so on...
- Pairs trading $\rightarrow$ group trading.
Why pairs work: the drunk and his dog

A metaphor adapted from [Murray, 1994] to the context of pairs trading.
A metaphor adapted from [Murray, 1994] to the context of pairs trading.

- A drunk customer sets out from the pub ("Gin Palace") and starts wandering in the streets (random walk, unit-root, integrated stochastic process).
Why pairs work: the drunk and his dog

A metaphor adapted from [Murray, 1994] to the context of pairs trading.

- A drunk customer sets out from the pub ("Gin Palace") and starts wandering in the streets (random walk, unit-root, integrated stochastic process).
- The accompanying dog always keeps track of the drunk’s position.
Why pairs work: the drunk and his dog

A metaphor adapted from [Murray, 1994] to the context of pairs trading.

- A drunk customer sets out from the pub ("Gin Palace") and starts wandering in the streets (random walk, unit-root, integrated stochastic process).
- The accompanying dog always keeps track of the drunk’s position.
- Whenever the drunk moves away, the dog will speed up to close the gap.
A metaphor adapted from [Murray, 1994] to the context of pairs trading.

- A drunk customer sets out from the pub (“Gin Palace”) and starts wandering in the streets (random walk, unit-root, integrated stochastic process).
- The accompanying dog always keeps track of the drunk’s position.
- Whenever the drunk moves away, the dog will speed up to close the gap.
- Whenever the drunk approaches too close, the dog will tend to widen the in-between distance.
The drunk and his dog: the story continues
Rory and Gary, two regular customers, look outside the pub’s window and bet on the drunk’s and the dog’s position.
Rory and Gary, two regular customers, look outside the pub’s window and bet on the drunk’s and the dog’s position. They observe the drunk and the dog individually but their course looks no different than a random walk (growing variance in location, lack of predictability).
Rory and Gary, two regular customers, look outside the pub’s window and bet on the drunk’s and the dog’s position.

They observe the drunk and the dog individually but their course looks no different than a random walk (growing variance in location, lack of predictability).

Suddenly, Gary throws the idea: “Well, it’s all a matter of finding the drunk, the dog should be around”.
Rory and Gary, two regular customers, look outside the pub’s window and bet on the drunk’s and the dog’s position.

They observe the drunk and the dog individually but their course looks no different than a random walk (growing variance in location, lack of predictability).

Suddenly, Gary throws the idea: “Well, it’s all a matter of finding the drunk, the dog should be around”.

He is right because no matter how far apart the two fellows may currently be, their courses will soon converge (co-integration).
Rory and Gary, two regular customers, look outside the pub’s window and bet on the drunk’s and the dog’s position.

They observe the drunk and the dog individually but their course looks no different than a random walk (growing variance in location, lack of predictability).

Suddenly, Gary throws the idea: “Well, it’s all a matter of finding the drunk, the dog should be around”.

He is right because no matter how far apart the two fellows may currently be, their courses will soon converge (co-integration).

Rory and Gary eventually agree to play the following game:
Rory and Gary, two regular customers, look outside the pub’s window and bet on the drunk’s and the dog’s position.

They observe the drunk and the dog individually but their course looks no different than a random walk (growing variance in location, lack of predictability).

Suddenly, Gary throws the idea: “Well, it’s all a matter of finding the drunk, the dog should be around”.

He is right because no matter how far apart the two fellows may currently be, their courses will soon converge (co-integration).

Rory and Gary eventually agree to play the following game:

“Why not betting on their relative distance rather than their absolute positions?”
Figure 1: Normalised price paths of Goodyear (GT) and Hewlett Packard (HPQ).
Why pairs trading is successful?

A behavioural-finance explanation:

- Investors quickly incorporate new information in security prices through their market positions.
- As a consequence, stock price movements reflect all publicly available information (future earnings prospects, corporate news, political events) (market efficiency).
- Fundamentally-related securities respond similarly to incoming news.
- Overreaction and herding behaviour of uninformed and “noisy” investors often drives prices apart.
- But, any deviation is temporary.
- In the long run, rational traders will likely correct mispricings and close the “gaps”.

---

See literature on “limits to arbitrage”.

Nikos S. Thomaidis, PhD  Statistical arbitrage and pairs trading
Basic steps in developing a pairs trading system

See also [Gatev et al., 2006].
Basic steps in developing a pairs trading system

See also [Gatev et al., 2006].

- **Group formation**
  - Pick fundamentally-related stocks and detect stable relative price relationships (*synthetic assets*).
Basic steps in developing a pairs trading system

- **Group formation**
  - Pick fundamentally-related stocks and detect stable relative price relationships (*synthetic assets*).

- **Group trading**
  - Determine the direction of the synthetic (divergence, re-convergence).
  - Market timing: when to open and close a trade on the synthetic.

*See also* [Gatev et al., 2006].
Basic steps in developing a pairs trading system

- **Group formation**
  - Pick fundamentally-related stocks and detect stable relative price relationships (*synthetic assets*).

- **Group trading**
  - Determine the direction of the synthetic (divergence, re-convergence).
  - Market timing: when to open and close a trade on the synthetic.

- **Risk management**
  - Minimise *divergence risk* (i.e. the risk that the gap between stocks further widens).
  - Fine-tune parameters with respect to a trading performance criterion (maximise expected return, maximise a reward-risk ratio, etc).

\[\text{See also [Gatev et al., 2006].}\]
Group formation strategy

Identify group-member stocks
- Normalised price distance
- Correlation in price series
- Event response
- Industry/sector classifications
- Liquidity criterion

Identify price relationships
- Stationarity (stability) criteria
- Capital allocation (floor and ceiling constraints)
Choose a charting time-frame.

Compute the correlation of historical price series, e.g.

<table>
<thead>
<tr>
<th>Pair</th>
<th>Stock 1</th>
<th>Stock 2</th>
<th>Stock 3</th>
<th>Stock 4</th>
<th>Stock 5</th>
<th>Stock 8</th>
<th>Stock 9</th>
<th>Stock 10</th>
<th>Correlation coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pair 1</td>
<td>Stock 1</td>
<td></td>
<td>Stock 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.91</td>
</tr>
<tr>
<td>Pair 2</td>
<td>Stock 1</td>
<td>Stock 2</td>
<td>Stock 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.87</td>
</tr>
<tr>
<td>Pair 3</td>
<td>Stock 2</td>
<td></td>
<td>Stock 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.81</td>
</tr>
<tr>
<td>Pair 4</td>
<td>Stock 8</td>
<td></td>
<td>Stock 10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.76</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>...</td>
</tr>
<tr>
<td>Pair 19</td>
<td>Stock 13</td>
<td></td>
<td>Stock 26</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.26</td>
</tr>
<tr>
<td>Pair 20</td>
<td>Stock 26</td>
<td></td>
<td>Stock 27</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.17</td>
</tr>
</tbody>
</table>

Pick the top 20% of pairs (i.e. 4 pairs) with the highest historical correlation.

Formed groups: \{1, 3, 5\}, \{2, 4\}, \{8, 10\}. 
Minimum normalised price distance (MNPD) criterion

- Quite popular in the literature
  [Gatev et al., 2006, Andrade et al., 2005].
- For each stock \( i = 1, 2, \ldots, N \), compute the *cumulative return* over the estimation sample period

\[
cr_{t,i} \equiv \prod_{\tau=1}^{t} (1 + r_{\tau,i}), \ t = 1, 2, \ldots, T
\]

where \( cr_{0,i} = 1 \) and \( r_{t,i} \) is the \( t \)-period’s return on stock \( i \).

- Introduce a “distance” measure:
  e.g. Euclidean distance

\[
d(i, j) \equiv |cr_{\star,i} - cr_{\star,j}| \equiv \sum_{t=1}^{T} (cr_{t,i} - cr_{t,j})^2
\]

- Rank stock pairs based on increasing values of \( d \) - pick the top \( a\% \) of the list for group formation.
Identify stationary relationships (1/5)


- Assume that a group of stocks with price vector \( \mathbf{P}_t = (P_{t1}, P_{t2}, \ldots, P_{tN})' \) satisfy the relationship:

  \[
P_{t1} = c + \beta_2 P_{t2} + \cdots + \beta_n P_{tN} + Z_t
  \]

  where \( Z_t \) is the *mispricing index* (captures temporal deviations from equilibrium).

- The coefficients of the relationship can be estimated using Ordinary Least Squares (OLS).
Construct a portfolio (*synthetic asset*) as follows:

<table>
<thead>
<tr>
<th>Stocks</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>⋅⋅⋅</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positions</td>
<td>+1</td>
<td>−(\hat{\beta}_2)</td>
<td>−(\hat{\beta}_3)</td>
<td>⋅⋅⋅</td>
<td>−(\hat{\beta}_N)</td>
</tr>
</tbody>
</table>

where \(\hat{\beta}_i\) is the OLS estimate of \(\beta_i\), computed over the formation period, and “+” ("-") indicates a *long* (*short*) position.

The portfolio value \(\hat{Z}_t \equiv \hat{\beta}' \cdot P_t\), where

\[
\hat{\beta} \equiv (1, -\hat{\beta}_2, -\hat{\beta}_3, \ldots, -\hat{\beta}_N)'
\]

is by construction mean-reverting (*fluctuates around \(\hat{c}\), the OLS estimate of \(c\)).
Identify relationships with OLS (3/5)
Identify relationships with OLS (4/5)

Stock 1

Stock 2

Equilibrium relationship:

\[ P_2 = 14.843 - 0.257 P_1 \]

actual price pairs

equilibrium relationship

positive mispricing

negative mispricing

Nikos S. Thomaidis, PhD
Statistical arbitrage and pairs trading
Identify relationships with OLS (5/5)

\[ Z_t = P_2 + 0.257 P_1 \]

- Stock 2 overpriced relative to Stock 1
- Stock 2 underpriced relative to Stock 1

Nikos S. Thomaidis, PhD
Statistical arbitrage and pairs trading
Conditions for meaningful capital allocations

- The money invested on each stock (average price $\times$ number of shares) must be on average below 80% and above 5% of the available capital.
- The ratio between the maximum and the minimum number of shares held from each asset should not exceed 10.
- and so on...

These place restrictions on the beta coefficients (stock holdings) $\rightarrow$ restricted OLS estimation
Group trading

Trading strategy

Specify the trading period
• Trading window
• Divergence criteria

Trading rules
• Divergence-convergence dynamics
• Threshold value
• Trade-open/trade-close points
Open a position in a group whenever the mispricing index exceeds a certain threshold:

- Buy the portfolio, if \( Z_t < \hat{Z}^L_t,\alpha_t \)
- Sell the portfolio, if \( Z_t > \hat{Z}^H_t,\alpha_t \)

where \( (\hat{Z}^L_t,\alpha_t, \hat{Z}^H_t,\alpha_t) \) is a 100 \times (1 - 2\alpha)\% confidence “envelope” on the value of the mispricing.
Open a position in a group whenever the mispricing index exceeds a certain threshold:

- Buy the portfolio, if \( Z_t < \hat{Z}_{t}^{L},\alpha \)
- Sell the portfolio, if \( Z_t > \hat{Z}_{t}^{H},\alpha \)

where \( (\hat{Z}_{t}^{L},\alpha , \hat{Z}_{t}^{H},\alpha ) \) is a \( 100 \times (1 - 2\alpha)\% \) confidence “envelope” on the value of the mispricing.

Unwind the position after \( h \) periods of time
Trading strategy (1/2)

- Open a position in a group whenever the mispricing index exceeds a certain threshold:
  - Buy the portfolio, if $Z_t < \hat{Z}_t^{L,\alpha}$
  - Sell the portfolio, if $Z_t > \hat{Z}_t^{H,\alpha}$

where $\left(\hat{Z}_t^{L,\alpha}, \hat{Z}_t^{H,\alpha}\right)$ is a $100 \times (1 - 2\alpha)\%$ confidence “envelope” on the value of the mispricing.

- Unwind the position after $h$ periods of time unless
Open a position in a group whenever the mispricing index exceeds a certain threshold:

- Buy the portfolio, if $Z_t < \hat{Z}_t^{L,\alpha}$
- Sell the portfolio, if $Z_t > \hat{Z}_t^{H,\alpha}$

where $(\hat{Z}_t^{L,\alpha}, \hat{Z}_t^{H,\alpha})$ is a $100 \times (1 - 2\alpha)\%$ confidence “envelope” on the value of the mispricing.

Unwind the position after $h$ periods of time unless the mispricing index continues to diverge (does not cross up the lower bound or cross down the upper bound).
Trading strategy (1/2)

- Open a position in a group whenever the mispricing index exceeds a certain threshold:
  - Buy the portfolio, if \( Z_t < \hat{Z}^L_t,\alpha \)
  - Sell the portfolio, if \( Z_t > \hat{Z}^H_t,\alpha \)

where \( (\hat{Z}^L_t,\alpha, \hat{Z}^H_t,\alpha) \) is a \( 100 \times (1 - 2\alpha) \)% confidence “envelope” on the value of the mispricing.

- Unwind the position after \( h \) periods of time unless the mispricing index continues to diverge (does not cross up the lower bound or cross down the upper bound).

- Close the position earlier and open a new position if the synthetic re-converges and crosses the opposite bound.
The two bounds \( \left( \hat{Z}_t^{L,\alpha}, \hat{Z}_t^{H,\alpha} \right) \) could be of the form \( \hat{c} \pm z_\alpha \hat{\sigma}_Z \).
The two bounds $\left( \hat{Z}_t^L, \hat{Z}_t^H \right)$ could be of the form $\hat{c} \pm z_\alpha \hat{\sigma}_Z$.

$\hat{c}$, $\hat{\sigma}_Z$ are the sample mean and standard deviation of the synthetic value over the formation period and $z_\alpha$ is a critical value from a $N(0,1)$ distribution.
The two bounds \( \left( \hat{Z}_{t}^{L,\alpha}, \hat{Z}_{t}^{H,\alpha} \right) \) could be of the form \( \hat{c} \pm z_{\alpha} \hat{\sigma}_Z \).

\( \hat{c}, \hat{\sigma}_Z \) are the sample mean and standard deviation of the synthetic value over the formation period and \( z_{\alpha} \) is a critical value from a \( N(0, 1) \) distribution.

This is the standard trade-triggering criterion adopted by most studies dealing with pairs trading (see e.g. [Andrade et al., 2005, Gatev et al., 2006]).
The two bounds \( \left( \hat{Z}_t^{L, \alpha}, \hat{Z}_t^{H, \alpha} \right) \) could be of the form \( \hat{c} \pm z_\alpha \hat{\sigma}_Z \).

\( \hat{c}, \hat{\sigma}_Z \) are the sample mean and standard deviation of the synthetic value over the formation period and \( z_\alpha \) is a critical value from a \( N(0, 1) \) distribution.

This is the standard trade-triggering criterion adopted by most studies dealing with pairs trading (see e.g. [Andrade et al., 2005, Gatev et al., 2006]).

Still, one could employ a more flexible criterion by dynamically-adjusting confidence bounds (see e.g. [Thomaidis et al., 2006, Thomaidis and Kondakis, 2012]).
Example: trading a group of 2 stocks (1/2)

Figure 3: Mispricing index: $Z_t = P_{GT} - 1.06 P_{HPQ}$, Trading parameters: Hold-out period ($HOP$) = 1 day, $\alpha_L = 10\%$, $\alpha_H = 5\%$. 

Nikos S. Thomaidis, PhD  Statistical arbitrage and pairs trading
Figure 4: HOP=1 day, $\alpha_L = 10\%$, $\alpha_H = 5\%$.
Figure 5: HOP=1 day, \( \alpha_L = 20\% \), \( \alpha_H = 20\% \).
Figure 6: HOP=1 day, $\alpha_L = 20\%$, $\alpha_H = 20\%$. 
System performance measurement

★ Are there parametrisations of the StatArb system that deliver consistently positive return?
★ Performance indicators (mean, std, downside std, information ratio (IR), downside IR).
★ How does the performance vary with market conditions, business/economic cycles?
★ Can superior returns be explained by exposure to systematic risks (market, industries, size factor, value factor, etc.)?
★ Are we capturing other patterns of stock movements (price reversals)?
★ How skillful is our system in terms of picking the right pairs/finding price equilibriums?
★ How able is our system to “sense” price deviations and predict the point of re-convergence?
★ Do our strategies involve too intense trading?

See also [Gatev et al., 2006].
Experimental setting

- Daily prices of 30 stock members of Dow Jones Industrial Average (DJIA) index (with dividends reinvested).\(^4\)
- Sample period: 3 Jan 1994 to 24 Feb 2010.
- Group formation:
  - Window length (WL) \{125, 250\} days.
  - Thin-trading criterion: disregard DJIA stocks that do not trade on a regular daily basis.
  - Choose matching stocks based on MNPD and MPC criteria (form groups from the 5\%, 20\% or 50\% highest-ranking pairs of the list).
- Trading strategy
  - Trading period: subsequent \{50, 125, 150\} days
  - Hold-out period (HOP): \{1, 5, 10, 25\} days
  - \(\alpha_L, \alpha_H \in \{1, 5, 10, 20, 40\}\%\)
- A total of 3,600 parametrisations.

\(^4\)Data downloaded from Yahoo!Finance as of 24 Feb 2010. See [Gatev et al., 2006, Andrade et al., 2005] for other examples of application designs.
## Best trading strategies

<table>
<thead>
<tr>
<th>Design parameters$^5$</th>
<th>Best strategy (Mean return)</th>
<th>Best strategy (IR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample: 1994-2010</td>
<td></td>
<td></td>
</tr>
<tr>
<td>WL</td>
<td>125</td>
<td>125</td>
</tr>
<tr>
<td>TP</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>GFC</td>
<td>MPC - 5%</td>
<td>MPC - 20%</td>
</tr>
<tr>
<td>HOP</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>$\alpha_L$ (%)</td>
<td>40</td>
<td>10</td>
</tr>
<tr>
<td>$\alpha_H$ (%)</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Performance of best trading strategies

<table>
<thead>
<tr>
<th>Trading measures</th>
<th>Best strategy (Mean return)</th>
<th>Best strategy (IR)</th>
<th>Buy &amp; hold portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sample: 1994-2010 (784 observations)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean(%)</td>
<td>11.65</td>
<td>7.78</td>
<td>5.92</td>
</tr>
<tr>
<td>Stdev(%)</td>
<td>26.44</td>
<td>9.94</td>
<td>22.00</td>
</tr>
<tr>
<td>DStdev(%)</td>
<td>23.75</td>
<td>6.48</td>
<td>16.54</td>
</tr>
<tr>
<td>IR</td>
<td>0.44</td>
<td>0.78</td>
<td>0.27</td>
</tr>
<tr>
<td>DIR</td>
<td>0.49</td>
<td>1.20</td>
<td>0.36</td>
</tr>
</tbody>
</table>

Table 1: Average weekly performance (annualised measures).
No portfolio manager would count on a single trading strategy for making money.
Portfolios of good strategies

- No portfolio manager would count on a single trading strategy for making money
- Mixing-up different parameter combinations
Portfolios of good strategies

- No portfolio manager would count on a single trading strategy for making money
- Mixing-up different parameter combinations
- “Bundles” of trading strategies

“Distribute your capital evenly between the top-a % of the parameterisations”
### Table 2: Average weekly performance on the full sample period (annualised measures).

<table>
<thead>
<tr>
<th>Strategies</th>
<th>Percentage of trading strategies</th>
<th>Best strategy</th>
<th>Buy &amp; hold</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100</td>
<td>90</td>
<td>65</td>
</tr>
<tr>
<td>Mean(%)</td>
<td>1.98</td>
<td>2.46</td>
<td>3.42</td>
</tr>
<tr>
<td>Stdev(%)</td>
<td>3.65</td>
<td>3.63</td>
<td>3.69</td>
</tr>
<tr>
<td>DStdev(%)</td>
<td>2.26</td>
<td>2.19</td>
<td>2.10</td>
</tr>
<tr>
<td>IR</td>
<td>0.54</td>
<td>0.68</td>
<td>0.93</td>
</tr>
<tr>
<td>DIR</td>
<td>0.88</td>
<td>1.12</td>
<td>1.63</td>
</tr>
</tbody>
</table>
### Table 3: Average weekly performance on the full sample period (annualised measures).

<table>
<thead>
<tr>
<th>Strategies</th>
<th>Percentage of trading strategies</th>
<th>Best strategy</th>
<th>Buy &amp; hold</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100</td>
<td>90</td>
<td>65</td>
</tr>
<tr>
<td>Mean(%)</td>
<td>1.98</td>
<td>2.46</td>
<td>3.42</td>
</tr>
<tr>
<td>Stdev(%)</td>
<td>3.65</td>
<td>3.63</td>
<td>3.66</td>
</tr>
<tr>
<td>DStdev(%)</td>
<td>2.26</td>
<td>2.19</td>
<td>2.09</td>
</tr>
<tr>
<td>IR</td>
<td>0.54</td>
<td>0.68</td>
<td>0.93</td>
</tr>
<tr>
<td>DIR</td>
<td>0.88</td>
<td>1.12</td>
<td>1.64</td>
</tr>
</tbody>
</table>
Performance of mixtures - Information ratio (2/2)

Dec95  Sep98  May01  Feb04  Nov06  Aug09

−50   0      50     100    150    200    250    300    350

Cumulative return (%)

IR-maximising strategies

top-100

top-90

top-65

top-35

top-10

best strategy

buy & hold

Nikos S. Thomaidis, PhD  Statistical arbitrage and pairs trading
Figure 7: Historical performance of the top-10% portfolio (IR) and systematic factors of risk. SMB and HML are the Fama-French size and value factors (available from http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).
<table>
<thead>
<tr>
<th>Strategies</th>
<th>Percentage of trading strategies</th>
<th>Best strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100</td>
<td>90</td>
</tr>
<tr>
<td>Alpha</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>MKT</td>
<td>-0.15</td>
<td>-0.15</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>SMB</td>
<td>0.00</td>
<td>-0.00</td>
</tr>
<tr>
<td></td>
<td>(0.83)</td>
<td>(0.89)</td>
</tr>
<tr>
<td>HML</td>
<td>-0.00</td>
<td>-0.00</td>
</tr>
<tr>
<td></td>
<td>(0.98)</td>
<td>(0.82)</td>
</tr>
<tr>
<td>MOM</td>
<td>-0.04</td>
<td>-0.04</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>LTR</td>
<td>-0.02</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>(0.36)</td>
<td>(0.52)</td>
</tr>
<tr>
<td>STR</td>
<td>0.04</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>Consumer Durables</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(0.95)</td>
<td>(0.96)</td>
</tr>
<tr>
<td>HiTec</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>Health</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>Other</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>(0.39)</td>
<td>(0.36)</td>
</tr>
</tbody>
</table>

Table 4: OLS estimates of the regression equation (*t*-statistics in parentheses).
Pairs trading is a cost-sensitive investment strategy.
Trading costs

- Pairs trading is a cost-sensitive investment strategy
- It involves
Pairs trading is a cost-sensitive investment strategy.
It involves:
- Frequent rebalancing of trading positions.
Pairs trading is a cost-sensitive investment strategy. It involves:
- Frequent rebalancing of trading positions
- Multiple openings and closings of trades
Trading costs

- Pairs trading is a cost-sensitive investment strategy.
- It involves:
  - Frequent rebalancing of trading positions.
  - Multiple openings and closings of trades.
  - Short-selling.
Pairs trading is a cost-sensitive investment strategy. It involves:
- Frequent rebalancing of trading positions
- Multiple openings and closings of trades
- Short-selling

Transaction costs, margin requirements, etc.
Trading costs

- Pairs trading is a cost-sensitive investment strategy
- It involves
  - Frequent rebalancing of trading positions
  - Multiple openings and closings of trades
  - Short-selling
- Transaction costs, margin requirements, etc
- How are the strategies expected to perform in a more realistic market environment?
Trading costs

- Pairs trading is a cost-sensitive investment strategy
- It involves
  - Frequent rebalancing of trading positions
  - Multiple openings and closings of trades
  - Short-selling
- Transaction costs, margin requirements, etc
- How are the strategies expected to perform in a more realistic market environment?
- Can generated profits offset trading costs?
### Top-10% (IR) portfolio of strategies

#### Sample period

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total days in sample:</td>
<td>4065</td>
</tr>
<tr>
<td>Total trading days in sample:</td>
<td>3865.7</td>
</tr>
<tr>
<td>Total number of traded stocks:</td>
<td>35</td>
</tr>
</tbody>
</table>

#### Group formation

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total number of formed groups:</td>
<td>71.43</td>
</tr>
<tr>
<td>Average size of groups:</td>
<td>4.51</td>
</tr>
</tbody>
</table>

#### Group trading

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total number of group openings during study:</td>
<td>195.76</td>
</tr>
<tr>
<td>Number of groups that never open:</td>
<td>4.19</td>
</tr>
<tr>
<td>Average number of active groups per trading day:</td>
<td>1.17</td>
</tr>
<tr>
<td>(Standard deviation)</td>
<td>0.45</td>
</tr>
<tr>
<td>Fraction of trading time groups are open:</td>
<td>0.88</td>
</tr>
<tr>
<td>Average number of times a group is opened over the trading period:</td>
<td>3.32</td>
</tr>
<tr>
<td>(Standard deviation)</td>
<td>2.24</td>
</tr>
<tr>
<td>Average duration of positions (days):</td>
<td>27.59</td>
</tr>
<tr>
<td>(Standard deviation)</td>
<td>28.94</td>
</tr>
<tr>
<td>Average duration of long positions (days):</td>
<td>24.50</td>
</tr>
<tr>
<td>(Standard deviation)</td>
<td>30.66</td>
</tr>
<tr>
<td>Average duration of short positions (days):</td>
<td>30.15</td>
</tr>
<tr>
<td>(Standard deviation)</td>
<td>27.05</td>
</tr>
</tbody>
</table>

**Notes:** (1) Averages over all parametrisations, (2) Standard deviation in parentheses.

See also [Sullivan et al., 1999, Andrade et al., 2005, Gatev et al., 2006] for descriptive measures of trading performance.
### Top-10% (IR) portfolio of strategies

<table>
<thead>
<tr>
<th>Divergence risk</th>
<th>Percentage of groups that are inactive (never open):</th>
<th>3.21</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Percentage of active groups with a single divergent trade:</td>
<td>26.31</td>
</tr>
<tr>
<td></td>
<td>Percentage of active groups that are opened and closed many time, though the final trade is divergent:</td>
<td>57.13</td>
</tr>
<tr>
<td></td>
<td>Percentage of active groups with no final divergent trade:</td>
<td>13.34</td>
</tr>
</tbody>
</table>

*Note: Averages over all 360 parametrisations (see also [Andrade et al., 2005, Do and Faff, 2010]).*
### The impact of transaction costs (1/2)

<table>
<thead>
<tr>
<th>Transaction cost</th>
<th>0 bps</th>
<th>10 bps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategies</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean(%)</td>
<td>5.93</td>
<td>5.38</td>
</tr>
<tr>
<td>Stdev(%)</td>
<td>4.31</td>
<td>4.34</td>
</tr>
<tr>
<td>DStdev(%)</td>
<td>2.55</td>
<td>2.54</td>
</tr>
<tr>
<td>IR</td>
<td>1.37</td>
<td>1.24</td>
</tr>
<tr>
<td>DIR</td>
<td>2.32</td>
<td>2.12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Transaction cost</th>
<th>50 bps</th>
<th>Buy &amp; hold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategies</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean(%)</td>
<td>4.93</td>
<td>5.92</td>
</tr>
<tr>
<td>Stdev(%)</td>
<td>4.33</td>
<td>22.00</td>
</tr>
<tr>
<td>DStdev(%)</td>
<td>2.55</td>
<td>16.54</td>
</tr>
<tr>
<td>IR</td>
<td>1.14</td>
<td>0.27</td>
</tr>
<tr>
<td>DIR</td>
<td>1.93</td>
<td>0.36</td>
</tr>
</tbody>
</table>

**Table 5:** Top-10% (IR) portfolio.

---

6 Fixed cost per unit of trading volume.
Figure 8: Historical performance of the top-10% (IR) portfolio assuming different levels of transaction costs.
Statistical arbitrage strategies are highly parametrised.
Statistical arbitrage strategies are highly parametrised.

Given a particular sample period and suppose we have enough time to experiment with alternative parametrisations of the system.
Statistical arbitrage strategies are highly parametrised.

Given a particular sample period and suppose we have enough time to experiment with alternative parametrisations of the system.

We will most likely be able to detect a strategy that beats the market, no matter what the performance measure is.
Statistical arbitrage strategies are highly parametrised.

Given a particular sample period and suppose we have enough time to experiment with alternative parametrisations of the system.

We will most likely be able to detect a strategy that beats the market, no matter what the performance measure is.

But, what can we say about the trading performance on a wider dataset?
Statistical arbitrage strategies are highly parametrised.

Given a particular sample period and suppose we have enough time to experiment with alternative parametrisations of the system.

We will most likely be able to detect a strategy that beats the market, \textit{no matter} what the performance measure is.

But, what can we say about the trading performance on a wider dataset?

\textbf{Data snooping ("dredging" or "fishing"):}

The practice of overfitting a trading strategy to a particular sample period [Sullivan et al., 1998, Sullivan et al., 1999, White, 2000].
Is the observed good performance of a StatArb system
Is the observed good performance of a StatArb system due to genuine superiority?
Is the observed good performance of a StatArb system

→ due to genuine superiority?

or…
Is the observed good performance of a StatArb system

→ due to genuine superiority?
  or...
→ due to a few lucky trades?
“Given enough computer time, we are sure that we can find a mechanical trading rule which ‘works’ on a table of random numbers, provided of course that we are allowed to test the rule on the same table of numbers which we used to discover the rule.” [Jensen and Bennington, 1970].

“Even when no exploitable [trading] model exists, looking long enough and hard enough at a given set of data will often reveal one or more [trading strategies] that look good, but are in fact useless.” [White, 2000].

“If you have 20,000 traders in the market, sure enough you’ll have someone who’s been up every day for the past few years and will show you a beautiful P&L. If you put enough monkeys on typewriters, one of the monkeys will write the Iliad in ancient Greek. But would you bet any money that he’s going to write the Odyssey next?” [Taleb, 1997].

---

How to eliminate data snooping biases?

See also [Lai and Xing, 2008], ch.11, for a discussion.
How to eliminate data snooping biases?

- Using an estimation and validation (test) data set
  - Helps measuring model performance beyond known data
  - Sensitive with respect to the particular choice of sample periods (training and testing)
  - Sensitive to market conditions

See also [Lai and Xing, 2008], ch.11, for a discussion.
How to eliminate data snooping biases?

- **Using an estimation and validation (test) data set\(^8\)**
  - Helps measuring model performance beyond known data
  - Sensitive with respect to the particular choice of sample periods (training and testing)
  - Sensitive to market conditions

- **Using multiple estimation/validation periods**
  - Performance assessments are more fair
  - Problems arise if these periods are consecutive
  - The choice of periods can introduce further bias

---

\(^8\)See also [Lai and Xing, 2008], ch.11, for a discussion.
How to eliminate data snooping biases?

- **Using an estimation and validation (test) data set**:  
  - Helps measuring model performance beyond known data  
  - Sensitive with respect to the particular choice of sample periods (training and testing)  
  - Sensitive to market conditions

- **Using multiple estimation/validation periods**:  
  - Performance assessments are more fair  
  - Problems arise if these periods are consecutive  
  - The choice of periods can introduce further bias

- **Statistical techniques**:  
  - Little sensitivity to market conditions  
  - Helps exploring new market scenarios (beyond those present in the dataset)

---

8See also [Lai and Xing, 2008], ch.11, for a discussion.
How would you choose your sample periods?

Nikos S. Thomaidis, PhD
Statistical arbitrage and pairs trading
Splitting the data set into estimation and validation periods

<table>
<thead>
<tr>
<th></th>
<th>Sample 1</th>
<th>Sample 2</th>
<th>Sample 3</th>
<th>Sample 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of observations (validation set)</td>
<td>756 days</td>
<td>756 days</td>
<td>756 days</td>
<td>1041 days</td>
</tr>
</tbody>
</table>
Trading performance comparisons (2/2)

Validation period 1

Validation set 2

Validation set 3

Validation set 4

Nikos S. Thomaidis, PhD
Statistical arbitrage and pairs trading
Random portfolios [Burns, 2006, Gatev et al., 2006]
How skillful is our strategy in terms of picking the *right* stocks
Random portfolios [Burns, 2006, Gatev et al., 2006]
How skillful is our strategy in terms of picking the *right* stocks at the *right* combination?
Statistical techniques

- **Random portfolios** [Burns, 2006, Gatev et al., 2006]
  How skillful is our strategy in terms of picking the *right* stocks at the *right* combination?

- **“Monkey” trading** [Perlin, 2009]
  Is our trading system superior to a “monkey”, which opens and closes trading positions at random points?
Random portfolios [Burns, 2006, Gatev et al., 2006]
How skillful is our strategy in terms of picking the right stocks at the right combination?

“Monkey” trading [Perlin, 2009]
Is our trading system superior to a “monkey”, which opens and closes trading positions at random points?

Other more sophisticated approaches

- Reality Check [White, 2000]
- Test of Superior Predictive Performance [Hansen, 2005]
- False discovery rate [Bajgrowiczy and Scailletz, 2009]
Figure 9: Skillful-picking days\textsuperscript{9}.

\textsuperscript{9}See also [Burns, 2006] for a probabilistic performance analysis.
Group-selection skills: interesting statistics

Based on the probability of “superiority”

- Percentage of skilled months: 63.10%
- Percentage of unskilled months: 36.90%
- Average number of consecutive skillful-picking months: 2.51
- Average number of consecutive unskilled-picking months: 1.47
Do stock-picking benefits accumulate over time?

Probability of outperformance: 98.20%
Is my trading system as smart as a monkey?

10 This particular monkey-trader was recruited from http://www.free-extras.com/images/monkey_thinking-236.htm.

Nikos S. Thomaidis, PhD
Statistical arbitrage and pairs trading
Skillful vs lucky trading

- Probability of superior group formation skills

- Monthly return

- Top-10% (IR) strategy

Nikos S. Thomaidis, PhD
Statistical arbitrage and pairs trading
Group-trading skills: interesting statistics

- Percentage of skilled months: 66.31%
- Percentage of unskilled months: 32.62%
- Average number of consecutive skilled months: 2.88
- Average number of consecutive unskilled months: 1.49
Beating the monkey in terms of cumulative return

Top−10% (IR) strategy

Probability of outperformance: 98.20%
How to improve your pairs trading system

Nikos S. Thomaidis, PhD

Statistical arbitrage and pairs trading
How to improve your pairs trading system

- Use firm fundamentals to select stocks with similar *elasticity* to risk factors.
How to improve your pairs trading system

- Use firm fundamentals to select stocks with similar elasticity to risk factors.
- Trade at higher frequencies (possibly using microstructure information).
How to improve your pairs trading system

- Use firm fundamentals to select stocks with similar *elasticity* to risk factors.
- Trade at higher frequencies (possibly using microstructure information).
- Select stocks with similar response patterns to market disturbances.
How to improve your pairs trading system

- Use firm fundamentals to select stocks with similar elasticity to risk factors.
- Trade at higher frequencies (possibly using microstructure information).
- Select stocks with similar response patterns to market disturbances
  → Event-response analysis ([Pole, 2007], section 2.4).
How to improve your pairs trading system

- Use firm fundamentals to select stocks with similar \textit{elasticity} to risk factors.
- Trade at higher frequencies (possibly using microstructure information).
- Select stocks with similar response patterns to market disturbances
  \[\rightarrow\] Event-response analysis ([Pole, 2007], section 2.4).
- Incorporate \textit{any} type of prior domain knowledge (e.g. industry or value/growth classifications).
Event-response analysis

Nikos S. Thomaidis, PhD  Statistical arbitrage and pairs trading
Pairs trading is a statistical arbitrage trading strategy.
Pairs trading is a **statistical** arbitrage trading strategy.

Performs better under limiting conditions:
Pairs trading is a statistical arbitrage trading strategy. It performs better under limiting conditions:

- infinitely-dimensional asset universe.
Pairs trading is a statistical arbitrage trading strategy.

- Performs better under limiting conditions:
  - infinitely-dimensional asset universe.
  - infinite amount of trading time, etc.
Epilogue

- Pairs trading is a **statistical** arbitrage trading strategy.
- Performs better under limiting conditions:
  - infinitely-dimensional asset universe.
  - infinite amount of trading time, etc.
- Computational challenges (processing huge amounts of information, asset selection, fine-tuning, model estimation).
Epilogue

Pairs trading is a **statistical** arbitrage trading strategy.

Performs better under limiting conditions:

- infinitely-dimensional asset universe.
- infinite amount of trading time, etc.

Computational challenges (processing huge amounts of information, asset selection, fine-tuning, model estimation).

Implementation challenges facing a real market environment (high portfolio turnover, market frictions).
Pairs trading is a statistical arbitrage trading strategy.

Performs better under limiting conditions:

- infinitely-dimensional asset universe.
- infinite amount of trading time, etc.

Computational challenges (processing huge amounts of information, asset selection, fine-tuning, model estimation).

Implementation challenges facing a real market environment (high portfolio turnover, market frictions).

If benefits (marginally) exceed costs your system is a hit!


References II


References IV


